CLASSIFICATION CONFIDENTIAL CONFIDENTIAL

CENTRAL INTELLIGENCE AGENCY
INFORMATION FROM
FOREIGN DOCUMENTS OR RADIO BROADCASTS

REPORT
CD NO.

50X1-HUM

| | | | |
|---|---|---|---|
| COUNTRY | USSR | DATE OF INFORMATION | 1950 |
| SUBJECT | Scientific - Mathematics, computers, approximations | DATE DIST. | 22 Jan 1951 |
| HOW PUBLISHED | Thrice-monthly periodical | | |
| WHERE PUBLISHED | Moscow/Leningrad | NO. OF PAGES | 4 |
| DATE PUBLISHED | 1 Oct 1950 | SUPPLEMENT TO REPORT NO. | |
| LANGUAGE | Russian | | |



THIS IS UNEVALUATED INFORMATION

SOURCE Doklady Akademii Nauk SSSR, Novaya Seriya, Vol LXXIV, No 4, 1950, pp 711-714.

# SYNTHESIS OF MATHEMATICAL MACHINES AND OF REAL OBJECTS

G. L. Polisar
Power Eng Inst imeni Krzhizhanovskiy
Acad Sci USSR
Submitted 29 Jul 1950 by Acad A. V. Vinter

/Numbers in parentheses refer to the bibliography./

We have already reported on the development of a new trend in the modeling of complex systems of interacting objects (1). In many cases (2) it is expedient to model only parts of the objects of a system and to study the dynamic states of systems which are made up of a combination of these models with the unsubstituted objects that remain. In a combination with unsubstituted objects, the models play the role of test stands.

A generalization and further development of the method where one employs the test stands as links in a closed dynamic system is the utilization of mathematical machines as universal models.

Just as differential equations which describe dynamic processes are a mathematical model of the investigated phenomenon, so can one consider devices that solve wide classes of these equations to be universal models in the widest sense of this word. Consequently, mathematical machines can serve as universal test stands, which are convenient for combining them in a dynamic state with real objects.

From this point of view, computing machines can be divided into two categories.

To the first category of universal computing devices of interest to us belong "continuous-action" /analog/ devices in which the role of the variables is played by natural quantities -- voltages, displacements, angle of rotation, etc.



CONFIDENTIAL

CLASSIFICATION CONFIDENTIAL

| STATE | X | NAVY | X | NSRB | | DISTRIBUTION | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ARMY | X | AIR | X | FBI | | | | | | | |

To this number belongs, for example, the mechanical differential analyzer. To the same category one can refer electromechanical and electronic equivalent schemes and electrical integrators of various types.

The use of computing, continuous-action devices as test stands has been described by the author in previous works (1, 2).

To the second category of universal computing devices belong various kinds of mathematical machines which solve, with an assigned approximation, systems of differential equations by way of execution of definite arithmetical operations or algorithms. Such devices permit one to set up a definite sequence of arithmetical operations according to one of the well-known methods. These devices repeat a definite cycle of operations, by causing at the output toward the end of each cycle an increment of the function which corresponds to an increment of the argument. Thus, these devices are computers of "discrete cyclical action" /digital/.

From the point of view of combining them with real objects it is necessary to take into consideration the tempo or speed of operation of such machines. One cycle corresponding to an increment $\Delta t$ of the independent variable can be finished by the mathematical machine in a time which is either less than $\Delta t$, equal to it, or greater.

As for the principles governing the operation of such machines, they can be quite different. Machines from which one requires great accuracy in the solutions ordinarily operate on the "computing-impulse" method /digital, discrete, flip-flop/. New complicated calculating machines of this type have already been put into use.

If a mathematical machine must be included as a model in an ordinary dynamic system with unsubstituted objects, then the time scale in the model must be such that, as in nature, one receives a respone of instantaneous values in the places where the model is combined with the objects in nature; or one must employ artificial methods for combining the unsubstituted objects and models with various time scales which would assure the possibility of investigating the dynamic states with an error not exceeding an assigned amount.

Let us consider, in principle, the possibility of combining models and unsubstituted objects under the condition that the time scales in them differ.

Let a "quadripole" /a 4-terminal network/ A, designate a model and let a quadripole B, correspond to an unsubstituted object. Let the parameters of the model (the mathematical machine) be such that the processes in it occur faster than in the object replaced by it.

Let the time scale in the model, A, be n times less than nature's time scale.

Let us divide the phenomenon under consideration into time intervals $\Delta t$. In the model this interval corresponds to the time interval $\Delta t/n$. At any moment of time let the model be connected up with the object. For concreteness, let the phenomena in the model be described by a system of ordinary differential equations with constant coefficients.

At the moment of contact the model obtains from the real object initial values of the dependent variables and the value of the so-called right part in the system of equations; and let these values of the right part be fixed by the aid of any means. The real object obtains the initial values of those functions by which the object is related to the model, by fixing these values in the time $\Delta t$. Afterward, the model is disconnected from the real object, and the phenomena in them are "untwisted apart" in the course of the time interval $\Delta t$. In the course of the time interval $\Delta t/n$ (which corresponds to $\Delta t$ in nature), the process is "cut off" in the model and the values of all the variables toward the end of this interval are fixed.

After this, toward the origin of the interval $\Delta_2 t$, the model again is connected up instantaneously with the real object, by interchanging with it the values of the transfer functions, etc. Such an operating state with cutoff makes it possible in principle to relate the model to the real object, in which the time scale is less than the natural one. Naturally, certain additional errors are introduced due to the breaking up into time intervals $\Delta t$, in the course of which we consider the right sides of the equation invariable, i.e., the functions which give the relation between the real objects and the models. The smaller the quantity $\Delta t$, the smaller the above-mentioned error.

The error of such a combination can be determined and compensated for.

Let us consider the solution by the indicated method of the problem:

$$\frac{du_1}{dt} = u_2 - u_1 \text{ and } \frac{du_2}{dt} = u_1 - u_2. \tag{1}$$

By actual solution the system will assume the following form:

$$\frac{dy_1}{dt} = y_2^{(\Delta t)} - y_1 \text{ and } \frac{dy_2}{dt} = y_1^{(\Delta t)} - y_2 \tag{2}$$

Subtracting (1) from (2), we obtain the error equation:

$$\frac{d\eta_1}{dt} = \eta_2 - \eta + y_2^{(\Delta t)} - y_2 \quad \& \quad \frac{d\eta_2}{dt} = \eta_1 - \eta_2 + y_1^{(\Delta t)} - y_1 \tag{3}$$

After integration we obtain:

$$\eta_1 + \eta_2 + \int_0^t (y_2^{(\Delta t)} - y_2)\, dx + \int_0^t (y_1^{(\Delta t)} - y_1)\, dx$$

$$\eta_2 - \eta_1 = e^{-2t} \int_0^t (y_1^{(\Delta t)} - y_1) e^{2x}\, dx - e^{-2t} \int_0^t (y_2^{(\Delta t)} - y_2)\, dx$$

Consideration of the integrals occurring in these equations at the moment of time $t = n\Delta t$ gives:

$$\int_0^t \left[ y_2^{(\Delta t)}(x) - y_2(x) \right] dx = -\frac{1}{2} \Delta t \left[ y_2(t) - y_2(0) \right] + \frac{2}{3} \theta (\Delta t)^2 M_2'' t$$

where $M_2'' = \max_{0 \leqslant x \leqslant t} |y_2''(x)|$.

In exactly the same manner:

$$\int_0^t \left[ y_1^{(\Delta t)}(x) - y_1(x) \right] dx = \frac{1}{2} \Delta t \left[ y_1(t) - y_1(0) \right] + \frac{2}{3} \theta (\Delta t)^2 M_1'' t$$

where $M_1'' = \max_{0 \leqslant x \leqslant t} |y_1''(x)|$

$$\int_0^t \left[ y_1^{(\Delta t)}(x) - y_1(x) \right] e^{2x} dx = \sum_{K=1}^{n} \int_{t_{K-1}}^{t_K} \left[ y_1^{(\Delta t)}(x) - y_1(x) \right] e^{2x} dx .$$

But for $t_{K-1} \leqslant x \leqslant t_n$ we have:

$$y^{(\Delta t)}(x) - y_1(x) = -(x - t_{K-1})\, y_1'(t_{K-1}) + \frac{1}{2} \theta_2 M_{1K}'' (x - t_{K-1})^2$$

$$e^{2x} = e^{2t_K} - (t_K - x)\, \theta_1 e^{2t_K} ;$$

[redacted] 50X1-HUM

therefore

$$\sum_{k=1}^{n}\int_{t_{k-1}}^{t_k}\left[y_1^{(\Delta t)}(x)-y_1(x)\right]e^{2x}dx=\frac{1}{2}\Delta t\sum_{k=1}^{n}y'(t_{k-1})e^{2t_k}\Delta t+\text{(con'd)}$$

$$\text{(con'd)}+\theta_1\frac{1}{6}(\Delta t)^2\sum_{k=1}^{n}y'(t_{k-1})/e^{2t_k}\,\Delta t+\theta_2\frac{1}{6}(\Delta t)^2\sum_{k=1}^{n}M''_{1k}\,e^{2t_k}\Delta t+\text{(con'd)}$$

$$\text{(con'd)}+\theta_1\theta_2\frac{(\Delta t)^3}{24}\sum_{k=1}^{n}e^{2t_k}M''_{1k}\Delta t.$$

Thus we obtain the evaluation required.

Let us consider the problem of error compensation. Let us write (2) in integral form:

$$y_1(t)=y_1(0)+\int_0^t\left[y_2(x)-y_1(x)\right]dx+\int_0^t\left[y_2^{(\Delta t)}(x)-y_2(x)\right]dx$$

$$y_2(t)=y_2(0)+\int_0^t\left[y_1(x)-y_2(x)\right]dx+\int_0^t\left[y_1^{(\Delta t)}(x)-y_1(x)\right]dx$$

But as we saw:

$$\int_0^t\left[y_2^{(\Delta t)}-y_2\right]dx=-\frac{1}{2}\Delta t\left[y_2(t)-y_2(0)\right]+k_2(\Delta t)^2$$

$$\int_0^t\left[y_1^{(\Delta t)}-y_1\right]dx=-\frac{1}{2}\Delta t\left[y_1(t)-y_2(0)\right]+k_1(\Delta t)^2 \quad \text{[sic]}$$

Therefore, by feeding in at the output, respectively, $y_1$ and $y_2$ at the moments of time $k \cdot \Delta t = t$ the quantities $\frac{1}{2}\Delta t\,[y_2(t) - y_2(0)]$ and $\frac{1}{2}\Delta t\,[y_1(t) - y_1(0)]$ we know the error in the solution to a quantity of the order $(\Delta t)^2$, i.e., we eliminate the main part of the error of the solution. The process of such compensation can be continued.

Thus, there exists in principle a possibility of combining the mathematical machines of discrete action with test objects for investigating the dynamic regimes of complex systems.

BIBLIOGRAPHY

1. G. L. Polisar, Tekhnika Vozdushnogo Flota, No 7, 1947, [redacted] 50X1-HUM

G. L. Polisar, DAN, Vol 61, No 4, 1948, [redacted]

2. G. L. Polisar, Research on Nonlinear Automatic Regulation Systems by the Method of Combination of Test Objects With Electrointegrators, Iz Ak Nauk SSSR, Otdel Tekh Nauk, No 3, 1949, pp 384-95.

- E N D -